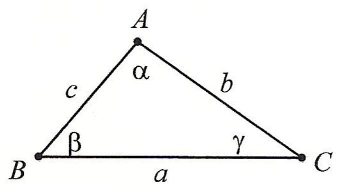
The Law of Sines

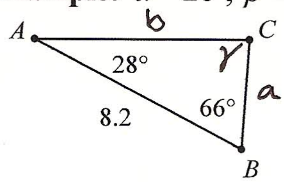
Solving a triangle means finding the measures of all the sides and angles. An **oblique triangle** is a triangle without a right angle. To solve an oblique triangle, we must know three pieces of information, at least one of which must be the length of a side. (Three angles define an infinite number of triangles).



The Law of Sines: In any triangle, $\dfrac{\sin\alpha}{a}=\dfrac{\sin\beta}{b}=\dfrac{\sin\gamma}{c}$

ASA or AAS: Find the third angle using the fact that the three angles of a triangle add to 180°. Then use the law of sines to find the other two sides of the triangle.

Example: $\alpha=28^\circ$, $\beta=66^\circ$, $c=8.2$



$\gamma = 180^\circ - (28^\circ + 66^\circ)$

$\boxed{\gamma = 86^\circ}$

$\dfrac{\sin 86^\circ}{8.2} = \dfrac{\sin 28^\circ}{a}$

$a = \dfrac{8.2 \sin 28^\circ}{\sin 86^\circ}$

$\boxed{a = 3.9}$

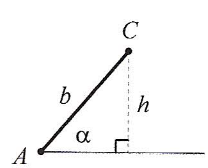
$\dfrac{\sin 86^\circ}{8.2} = \dfrac{\sin 66^\circ}{b}$

$b = \dfrac{8.2 \sin 66^\circ}{\sin 86^\circ}$

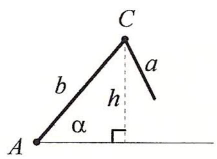
$\boxed{b = 7.5}$

SSA (The Ambiguous Case): If you know two sides and a non-included angle (an angle that is not between the sides), there may be zero, one, or two possible triangles that fit the given measurements. To figure out how many triangles there are for an *acute* angle α, do the following:

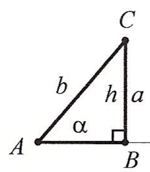
1. Draw the given angle (α) in standard position with a terminal side of length b. Don't draw side a yet.
2. Let h be an altitude from C to the initial side of α.



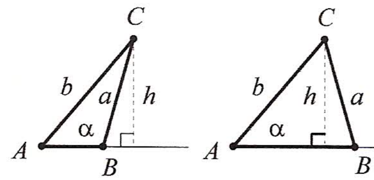
3. Since $\sin\alpha = h/b$, $h = b\sin\alpha$. Compare h to a as follows:
   a. If $a < h$, then no triangle can be formed. opp side < height ⇒ no △s
   b. If $a = h$, then one triangle (a right triangle) can be formed. opp side = height ⇒ rt. △
   c. If $h < a < b$, then two triangles can be formed. opp. side between adj. side & height ⇒ 2 △s
   d. If $a \ge b$, then one triangle can be formed. → opp side ≥ adj. side ⇒ one △



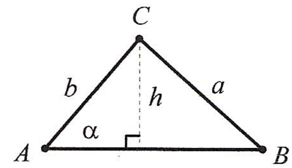
(a) $a<h$
no triangle



(b) $a=h$
one triangle



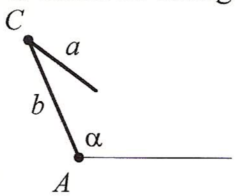
(c) $h<a<b$
two triangles



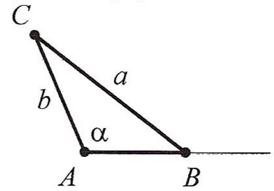
(d) $a \geq b$
one triangle

If α is *obtuse*, there is either no triangle (if $a \leq b$) or one triangle (if $a > b$) possible.

(a) $a \leq b$
no triangle



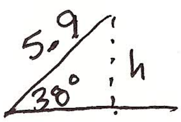
(b) $a > b$
one triangle



Examples:

a) $\beta = 38°$, $b = 2.9$, $c = 5.9$

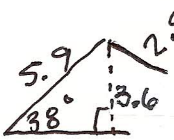
c is the side next to β
b is across from β



$\sin 38° = \frac{h}{5.9}$

$h = 3.6$

$\boxed{\text{no triangle}}$



$2.9 < 3.6$
opposite side not long enough to reach base.

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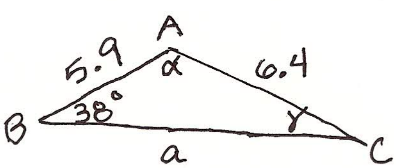
b) $\beta = 38°$, $b = 6.4$, $c = 5.9$ opp side longer than adjacent side ⇒ one Δ



$\frac{\sin 38°}{6.4} = \frac{\sin \gamma}{5.9}$

$\frac{5.9 \sin 38°}{6.4} = \sin \gamma$

$\sin \gamma = .5676$

$\boxed{\gamma = 34.6°}$

$\alpha = 180° - (38° + 34.6°)$ $\boxed{\alpha = 107.4°}$

$\frac{\sin 38°}{6.4} = \frac{\sin 107.4°}{a}$

$\frac{6.4 \sin 107.4°}{\sin 38°} = a$ $\boxed{a = 9.9}$

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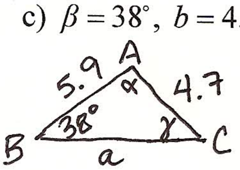
c) $\beta = 38°$, $b = 4.7$, $c = 5.9$

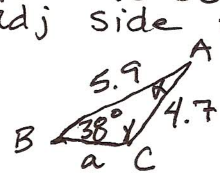
$h = 3.6$ (see example a)

opp. side between height & adj side ⇒ 2 Δs



$\frac{\sin 38°}{4.7} = \frac{\sin \gamma}{5.9}$

$\sin \gamma = .773$



$\boxed{\gamma = 50.6°}$

$\alpha = 180° - (38° + 50.6°)$ $\boxed{\alpha = 91.4°}$

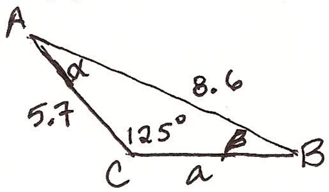
$\frac{\sin 38°}{4.7} = \frac{\sin 91.4°}{a}$ $\boxed{a = 7.6}$

$\gamma = 180° - 50.6°$ $\boxed{\gamma = 129.4°}$

$\alpha = 180° - (38° + 129.4°)$ $\boxed{\alpha = 12.6°}$

$\frac{\sin 38°}{4.7} = \frac{\sin 12.6°}{a}$ $\boxed{a = 1.7}$

d) $\gamma = 125^\circ$, $b = 5.7$, $c = 8.6$



obtuse ∠ &

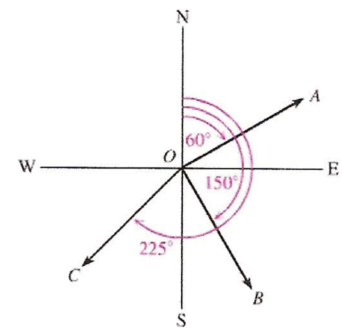
side opp > side adj ⇒ 1 Δ

$$\frac{\sin 125^\circ}{8.6} = \frac{\sin \beta}{5.7} \qquad \sin \beta = .543 \qquad \boxed{\beta = 32.9^\circ}$$

$$\alpha = 180^\circ - (125^\circ + 32.9^\circ) \qquad \boxed{\alpha = 22.1^\circ}$$

$$\frac{\sin 125^\circ}{8.6} = \frac{\sin 22.1^\circ}{a} \qquad \boxed{a = 3.9}$$

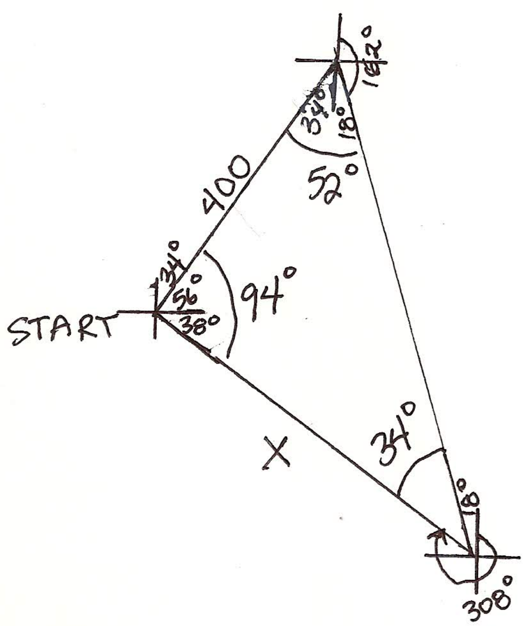
Bearing: The measure of an angle that describes the direction of a ray is called the bearing. Bearing is the clockwise angle from due north.



Another way to express bearing is to describe the acute angle that the ray makes with a ray pointing due north or south. For example:

N60°E is a bearing of 60° east of north
S30°E is a bearing of 30° east of south
S45°W is a bearing of 45° west of south

Example: During an important NATO exercise, an F-14 Tomcat left the carrier Nimitz on a course with a bearing of 34° and flew 400 miles. Then the F-14 flew for some distance on a course with a bearing of 162°. Finally, the plane flew back to its starting point on a course with a bearing of 308°. What distance did the plane fly on the final leg of the journey? Round to the nearest tenth of a mile.

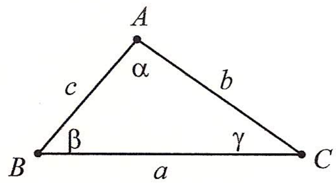


$$\frac{\sin 34^\circ}{400} = \frac{\sin 52^\circ}{x}$$

$$\frac{400 \sin 52^\circ}{\sin 34^\circ} = x$$

$$x = \boxed{563.7 \text{ mi}}$$

The Law of Cosines

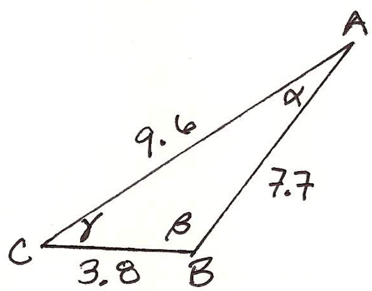


The Law of Cosines: In any triangle,
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

SSS: Use the fact that the largest angle is across from the longest side of the triangle to solve for the largest angle using the law of cosines. (For example, if c is the longest side, use the equation $c^2 = a^2 + b^2 - 2ab\cos\gamma$ to solve for γ.) Then use the law of sines to find the remaining angles, which will both be acute.

Example: $a = 3.8$, $b = 9.6$, $c = 7.7$

Largest ∠ is β - Find that first



$9.6^2 = 3.8^2 + 7.7^2 - 2(3.8)(7.7)\cos\beta$

$92.16 = 73.73 - 58.52\cos\beta$

$18.43 = -58.52\cos\beta$

$\cos\beta = -.3149$

$\boxed{\beta = 108.4°}$

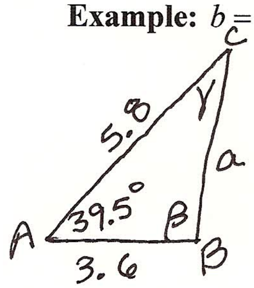
$\dfrac{\sin 108.4°}{9.6} = \dfrac{\sin\alpha}{3.8}$

$\sin\alpha = .376$ $\boxed{\alpha = 22.1°}$

$\gamma = 180° - (108.4° + 22.1°)$ $\boxed{\gamma = 49.5°}$

SAS: Find the length of the third side using the law of cosines. Use the law of sines to find the angle across from the shorter of the two given sides. Find the remaining angle by subtracting the first two from 180°.

Example: $b = 5.8$, $c = 3.6$, $\alpha = 39.5°$



$a^2 = 3.6^2 + 5.8^2 - 2(3.6)(5.8)\cos 39.5°$

$a^2 = 14.38$ $\boxed{a = 3.8}$

γ is smallest ∠, so it must be acute. Find it first.

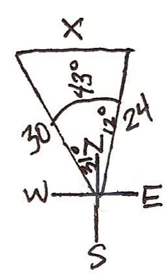
$\dfrac{\sin 39.5°}{3.8} = \dfrac{\sin\gamma}{3.6}$ $\sin\gamma = .603$ $\boxed{\gamma = 37.1°}$

$\beta = 180° - (39.5° + 37.1°)$ $\boxed{\beta = 103.4°}$

Example: Jan and Dean started hiking from the same location at the same time. Jan hiked at 4 mph with bearing N12°E, and Dean hiked at 5 mph with bearing N31°W. How far apart were they after 6 hours? Round to the nearest tenth of a mile.

4 mph for 6 hr = 24 mi

5 mph for 6 hr = 30 mi

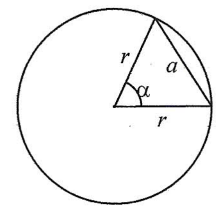


$x^2 = 30^2 + 24^2 - 2(30)(24)\cos 43°$

$x^2 = 422.85$

$x = \boxed{20.6 \text{ mi}}$

Length of a Chord: If a chord of length a is intercepted by a central angle α in a circle of radius r, then $a = r\sqrt{2-2\cos\alpha}$. (This formula is derived from the law of cosines.)



Example: Find the length of the chord intercepted by a central angle of 33.8° in a circle of radius 22.4 ft.

$\alpha = 33.8°$

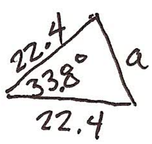
$r = 22.4$ ft

$a = ?$

$a = 22.4\sqrt{2-2\cos 33.8°}$

$\boxed{a = 13.0 \text{ ft}}$

-OR-



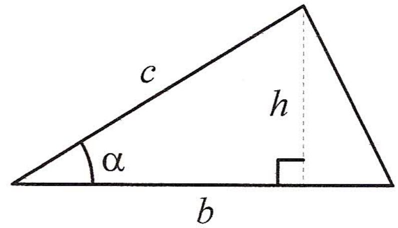
$a^2 = 22.4^2 + 22.4^2 - 2(22.4)(22.4)\cos 33.8°$

$a^2 = 169.61$

$\boxed{a = 13.0 \text{ ft}}$

Area of a Triangle

The formula $A=\frac{1}{2}bh$ gives the area of a triangle.



In the diagram at the right, $\sin\alpha=\frac{h}{c}$, so $h=c\sin\alpha$. Using substitution, we derive the formula $A=\frac{1}{2}bc\sin\alpha$. Depending on which angles and sides are known, the formulas $A=\frac{1}{2}ac\sin\beta$ and $A=\frac{1}{2}ab\sin\gamma$ can also be used.

Use two sides & angle between them

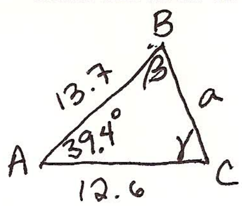
Using the law of cosines, it is possible to derive a formula for the area of a triangle that involves only the lengths of the sides of the triangle. The formula is known as "Heron's Formula" after Heron of Alexandria, who is believed to have discovered it around AD 75.

Heron's Formula: The area of a triangle with sides of lengths *a*, *b*, and *c* is given by:

$$A=\sqrt{S(S-a)(S-b)(S-c)}, \text{ where } S=(a+b+c)/2.$$

Examples:

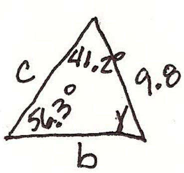
Find the area of the triangle with $\alpha=39.4°$, $b=12.6$, and $c=13.7$



$A=\frac{1}{2}(12.6)(13.7)\sin 39.4°$

$\boxed{A=54.8}$

Find the area of a triangle with $\alpha=56.3°$, $\beta=41.2°$, and $a=9.8$



$\frac{\sin 56.3°}{9.8}=\frac{\sin 41.2°}{b}$ $\quad b=7.8$

$\gamma=180°-(41.2°+56.3°)$ $\quad \gamma=82.5°$

$A=\frac{1}{2}(9.8)(7.8)\sin 82.5°=\boxed{37.9}$

Find the area of the triangle with $a=12$, $b=8$, and $c=6$

$S=\frac{12+8+6}{2}=13$

$A=\sqrt{13(13-12)(13-8)(13-6)}=\boxed{21.3}$

Find the area of a triangle with $a=346$, $b=234$, and $c=422$

$S=\frac{346+234+422}{2}=501$

$A=\sqrt{501(501-346)(501-234)(501-422)}=\boxed{40,471.9}$

Vectors

Scalar Quantities: Quantities such as length, area, volume, temperature, and time, which have magnitude (size), but no direction.

Vector Quantities: Quantities that involve both a magnitude and direction, such as velocity, acceleration, and force. These quantities can be represented by **directed line segments** called **vectors.**

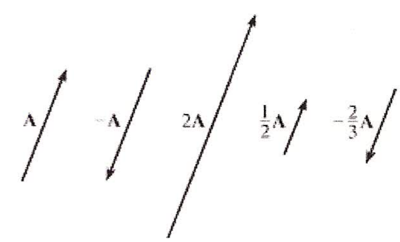
The length of a vector represents the **magnitude** of the vector quantity. The **direction** is indicated by the position of the vector and the arrowhead at one end.

Notation: $\overrightarrow{AB}$ is used to name a vector with **initial point** A and **terminal point** B. Vectors may also be denoted by bold letters. $\overrightarrow{AB}$ can also be written as **AB**. If the initial and terminal points are not specified, vectors can be named by a single uppercase or lowercase letter (eg. $\vec{b}$, $\vec{B}$, **b**, or **B**.) The magnitude of vector **A** is written $|\mathbf{A}|$.

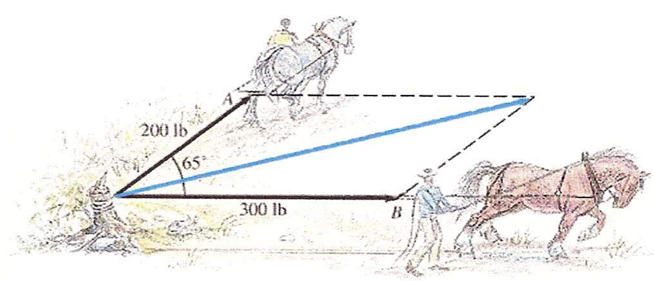
Equal Vectors: Vectors with the same magnitude and direction. They do not have to be in the same place.

Zero Vector: A vector with no magnitude and no direction. It is denoted by **0**.

Scalar Multiplication: For any scalar k and vector **A,** k**A** is a vector with magnitude $|k|$ times the magnitude of **A**. If $k > 0$, then the direction of k**A** is the same as the direction of **A**. If $k < 0$, the direction of k**A** is opposite to the direction of **A**. If $k = 0$, then $k\mathbf{A} = \mathbf{0}$.

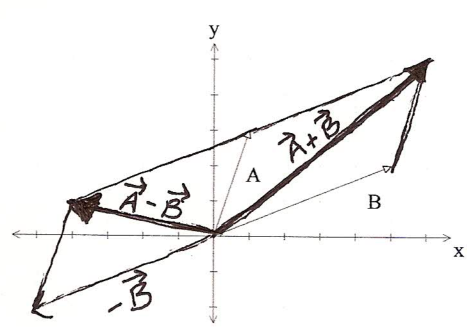


Suppose two draft horses are pulling on a tree stump with forces of 200 pounds and 300 pounds, with an angle of 65° between the forces. If **A** and **B** had the same direction, then there would be a total force of 500 pounds acting on the stump, but the total force is less because of the angle between the forces. By the **parallelogram law**, the force acting along the diagonal of the parallelogram, with a magnitude equal to the length of the diagonal, has the same effect on the stump as the two forces **A** and **B**. The force **A** + **B** acting along the diagonal is called the **sum** or **resultant** of A and **B**.

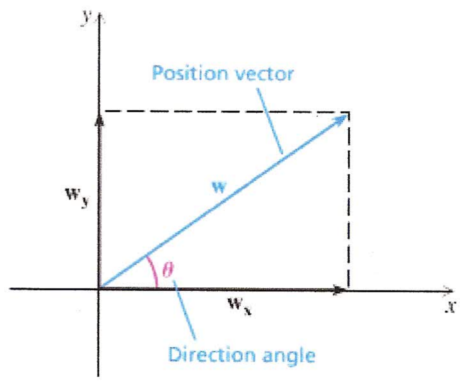


Vector Addition: To find the resultant or sum **A** + **B** of any vectors **A** and **B**, position **B** (without changing its magnitude or direction) so that the initial point of **B** coincides with the terminal point of **A**. The vector that begins at the initial point of **A** and ends at the terminal point of **B** is the vector **A** + **B**. For every vector **A**, there is a vector – **A**, with the same magnitude as **A**, but the opposite direction. For any two vectors **A** and **B**, **A** – **B** = **A** + (– **B**).

Example: Sketch the vectors **A** + **B** and **A** – **B**.



Any nonzero vector **w** is the sum of a **horizontal component, $\mathbf{w}_x$**, and a **vertical component, $\mathbf{w}_y$**. If a vector **w** is placed in a rectangular coordinate system so that its initial point is the origin, then **w** is called a **position vector**. The angle θ formed by the positive x-axis and a position vector is the **direction angle** for the position vector.



If the vector **w** has magnitude r, direction angle θ, horizontal component $\mathbf{w}_x$, and vertical component $\mathbf{w}_y$, then we get

$\cos\theta = \frac{|\mathbf{w}_x|}{r}$ and $\sin\theta = \frac{|\mathbf{w}_y|}{r}$ or $|\mathbf{w}_x| = |r\cos\theta|$ and $|\mathbf{w}_y| = |r\sin\theta|$.

Examples: Find the magnitude of the horizontal and vertical components for each vector **v** with the given magnitude and direction angle θ. Round to the nearest tenth.

a) $|\mathbf{v}| = 5.6,\ \theta = 22°$

$|\vec{v}_x| = |5.6 \cos 22°| = \boxed{5.2}$

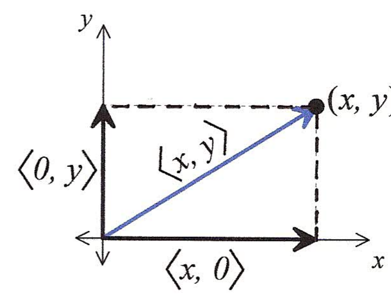
$|\vec{v}_y| = |5.6 \sin 22°| = \boxed{2.1}$

b) $|\mathbf{v}| = 445,\ \theta = 211.1°$

$|\vec{v}_x| = |445 \cos 211.1°| = \boxed{381.0}$

$|\vec{v}_y| = |445 \sin 211.1°| = \boxed{229.9}$

Component Form: The notation $\langle x, y\rangle$ is used to define a position vector with terminal point (x, y). This is called component form because the horizontal component is $\langle x, 0\rangle$ and its vertical component is $\langle 0, y\rangle$.



The magnitude of the vector $\mathbf{v} = \langle x, y\rangle$ is $|\mathbf{v}| = r = \sqrt{x^2 + y^2}$. To find the direction angle, use $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$.

If a vector has magnitude r and direction angle θ, its component form is $\langle r\cos\theta,\ r\sin\theta\rangle$.

Examples: Find the magnitude and direction angle of each vector.

a) $\mathbf{v} = \langle 2, -6\rangle$ QIV

$|\vec{v}| = \sqrt{2^2 + (-6)^2} = \sqrt{40} = \boxed{2\sqrt{10}}$

$\tan\theta = \frac{-6}{2} = -3$

$\theta = -71.6°$

$-71.6° + 360° = \boxed{288.4°}$

b) $\mathbf{v} = \langle -3, 2\rangle$ QII

$|\vec{v}| = \sqrt{(-3)^2 + 2^2} = \boxed{\sqrt{13}}$

$\cos\theta = \frac{-3}{\sqrt{13}}$

$\boxed{\theta = 146.3°}$

Examples: Find the component form for each vector **v** with the given magnitude and direction angle θ.

a) $|\mathbf{v}| = 12,\ \theta = 120°$

$x = 12 \cos 120° = 12(-\tfrac{1}{2}) = -6$

$y = 12 \sin 120° = 12(\tfrac{\sqrt{3}}{2}) = 6\sqrt{3}$

$\boxed{\langle -6, 6\sqrt{3}\rangle}$

b) $|\mathbf{v}| = 50,\ \theta = 120°$

$x = 50 \cos 120° = 50(-\tfrac{1}{2}) = -25$

$y = 50 \sin 120° = 50(\tfrac{\sqrt{3}}{2}) = 25\sqrt{3}$

$\boxed{\langle -25, 25\sqrt{3}\rangle}$

If $\mathbf{A} = \langle a_1, a_2 \rangle$, $\mathbf{B} = \langle b_1, b_2 \rangle$, and k is a scalar, then

Vector Arithmetic:

1. $k\mathbf{A} = \langle ka_1, ka_2 \rangle$ **Scalar Product**
2. $\mathbf{A} + \mathbf{B} = \langle a_1 + b_1, a_2 + b_2 \rangle$ **Vector Sum**
3. $\mathbf{A} - \mathbf{B} = \langle a_1 - b_1, a_2 - b_2 \rangle$ **Vector Difference**
4. $\mathbf{A} \cdot \mathbf{B} = a_1 b_1 + a_2 b_2$ **Dot Product**

Examples: Let $\mathbf{w} = \langle -1, -3 \rangle$ and $\mathbf{v} = \langle 3, -4 \rangle$. Perform the operations indicated.

a) $\mathbf{w} - \mathbf{v}$

$\langle -1, -3 \rangle - \langle 3, -4 \rangle$

$= \boxed{\langle -4, 1 \rangle}$

b) $-8\mathbf{v}$

$-8\langle 3, -4 \rangle$

$= \boxed{\langle -24, 32 \rangle}$

c) $3\mathbf{w} + 4\mathbf{v}$

$3\langle -1, -3 \rangle + 4\langle 3, -4 \rangle$

$= \langle -3, -9 \rangle + \langle 12, -16 \rangle$

$= \boxed{\langle 9, -25 \rangle}$

d) $\mathbf{w} \cdot \mathbf{v}$

$\langle -1, -3 \rangle \cdot \langle 3, -4 \rangle$

$= (-1)(3) + (-3)(-4)$

$= -3 + 12 = \boxed{9}$

The Angle Between Two Vectors:

If $\mathbf{A}$ and $\mathbf{B}$ are nonzero vectors and α is the smallest positive angle between them, then $\cos\alpha = \dfrac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$.

Examples: Find the smallest positive angle between the following vectors:

a) $\langle 1, 3 \rangle$ and $\langle 5, 2 \rangle$

$\langle 1, 3 \rangle \cdot \langle 5, 2 \rangle = (1)(5) + (3)(2) = 11$

$|\langle 1, 3 \rangle| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$|\langle 5, 2 \rangle| = \sqrt{5^2 + 2^2} = \sqrt{29}$

$\cos\alpha = \dfrac{11}{\sqrt{10}\sqrt{29}} = .646$ $\boxed{\alpha = 49.8^\circ}$

b) $\langle -1, 5 \rangle$ and $\langle 2, 7 \rangle$

$\langle -1, 5 \rangle \cdot \langle 2, 7 \rangle = (-1)(2) + (5)(7) = 33$

$|\langle -1, 5 \rangle| = \sqrt{(-1)^2 + 5^2} = \sqrt{26}$

$|\langle 2, 7 \rangle| = \sqrt{2^2 + 7^2} = \sqrt{53}$

$\cos\alpha = \dfrac{33}{\sqrt{26}\sqrt{53}} = .889$ $\boxed{\alpha = 27.3^\circ}$

Parallel Vectors: The vectors $\mathbf{A}$ and $\mathbf{B}$ are parallel if and only if $\mathbf{A} = k\mathbf{B}$ for a nonzero scalar k.

Perpendicular Vectors: The vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular if and only if $\mathbf{A} \cdot \mathbf{B} = 0$.

Examples: Determine whether each pair of vectors is parallel, perpendicular, or neither.

a) $\langle -2, 3 \rangle$ and $\langle 6, 4 \rangle$

$\langle -2, 3 \rangle \cdot \langle 6, 4 \rangle$

$= (-2)(6) + (3)(4) = 0$

$\boxed{\text{perpendicular}}$

b) $\langle 2, -5 \rangle$ and $\langle -4, 10 \rangle$

$\langle -4, 10 \rangle = -2\langle 2, -5 \rangle$

$\boxed{\text{parallel}}$

c) $\langle 2, 6 \rangle$ and $\langle 6, 2 \rangle$

$\langle 2, 6 \rangle \cdot \langle 6, 2 \rangle$

$= 2(6) + (6)(2) = 24$

$\boxed{\text{neither}}$

The vectors $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$ are called **unit vectors** because each has magnitude one. For any vector $\langle a_1, a_2 \rangle$, we have $\langle a_1, a_2 \rangle = a_1 \langle 1, 0 \rangle + a_2 \langle 0, 1 \rangle = a_1\mathbf{i} + a_2\mathbf{j}$. The form $a_1\mathbf{i} + a_2\mathbf{j}$ is called a **linear combination** of the vectors $\mathbf{i}$ and $\mathbf{j}$.

Examples: Write each vector as a linear combination of the unit vectors $\mathbf{i}$ and $\mathbf{j}$.

a) $\mathbf{A} = \langle -1, 7 \rangle$

$\vec{A} = -\vec{i} + 7\vec{j}$

b) $\mathbf{B} = \langle 0, -9 \rangle$

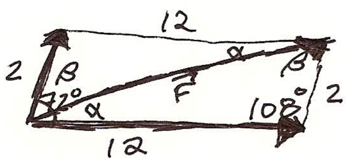
$\vec{B} = -9\vec{j}$

Math 1060 – 5.5 Notes

Applications of Vectors

Example: Forces of 2 lb and 12 lb are acting at an angle of 72° to each other. Find the magnitude of the resultant force and the angle between the resultant and each force.

Consecutive ∠s of parallelograms are supplementary.



$$|\vec{F}|^2 = 12^2 + 2^2 - 2(12)(2)\cos 108°$$

$$|\vec{F}|^2 = 162.8$$

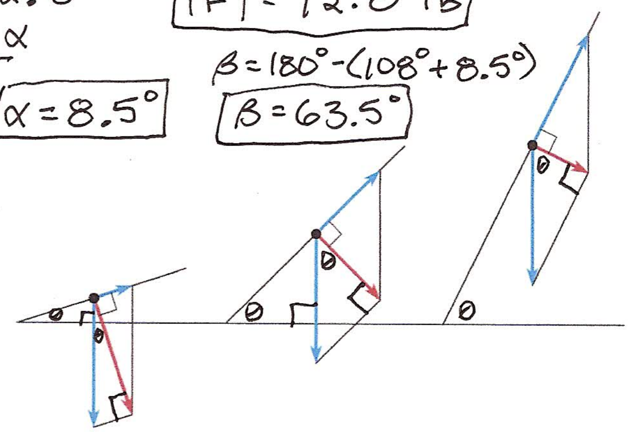
$$\boxed{|\vec{F}| = 12.8 \text{ lb}}$$

$$\frac{\sin 108°}{12.8} = \frac{\sin \alpha}{2}$$

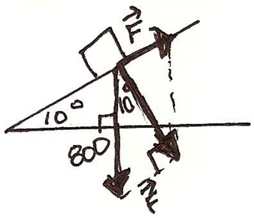
$$\sin \alpha = 0.1486 \quad \boxed{\alpha = 8.5°}$$

$$\beta = 180° - (108° + 8.5°) \quad \boxed{\beta = 63.5°}$$

Inclined Plane Problems: The weight of an object is always modeled as a vertical vector and the force required to move the object is modeled as a vector parallel to the inclined plane. Its length increases as the incline increases. The resultant of these two forces is a vector perpendicular to the inclined plane. It is what a bathroom scale would read if trapped between the object and the plane.



Example: Find the amount of force required to push an 800-pound block of ice up a ramp that is inclined 10°.

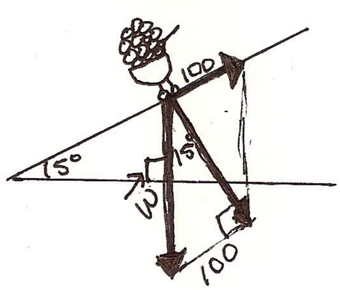


$$\sin 10° = \frac{|\vec{F}|}{800}$$

$$|\vec{F}| = 800 \sin 10°$$

$$= \boxed{138.9 \text{ lb}}$$

Example: A landscaper uses 100 pounds of force to pull a cart full of rocks up a driveway that is inclined 15°. What is the weight of the cart?

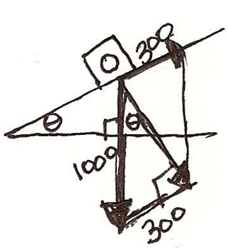


$$\sin 15° = \frac{100}{|\vec{w}|}$$

$$|\vec{w}| = \frac{100}{\sin 15°}$$

$$|\vec{w}| = \boxed{386.4 \text{ lb}}$$

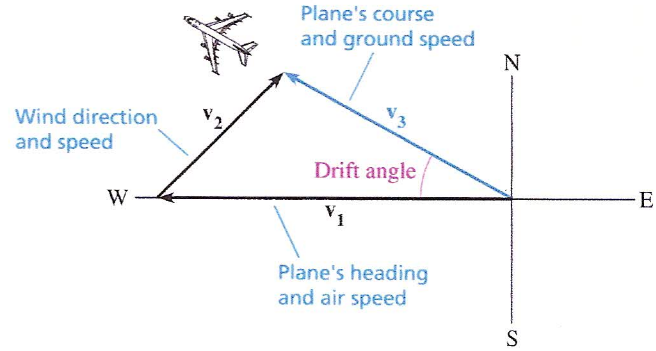
Example: If 300 pounds of force is required to push a 1000-pound safe up a ramp, then what is the angle of inclination of the ramp?



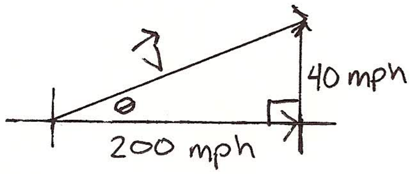
$$\sin \theta = \frac{300}{1000}$$

$$\theta = \boxed{17.5°}$$

Navigation Problems: Wind affects the speed and direction of a plane. The **heading** and **air speed** are the direction and speed of the plane before wind is taken into account. The **course** and **ground speed** are the direction and speed of the plane with wind taken into account. The angle between the heading and the course is the **drift angle**.



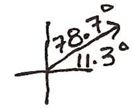
Example: An airplane is headed due east with an air speed of 200 mph. The wind is out of the south (bearing 0°) at 40 mph. Find the drift angle, the ground speed, and the course of the airplane.



$|\vec{v}|^2 = 40^2 + 200^2$

$|\vec{v}|^2 = 41{,}600$

$|\vec{v}| = \boxed{204.0 \text{ mph}}$ ← ground speed



$\tan\theta = \dfrac{40}{200}$

$\theta = \boxed{11.3°}$ ← drift angle

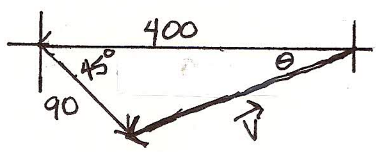
course: $\boxed{78.7°}$

Example: An airplane is headed due west with an air speed of 400 mph. The wind is out of the northwest (bearing 135°) at 90 mph. Find the drift angle, the ground speed and the course of the airplane.

$|\vec{v}|^2 = 90^2 + 400^2 - 2(90)(400)\cos 45°$

$|\vec{v}|^2 = 117{,}188$

$|\vec{v}| = \boxed{342.3 \text{ mph}}$ ← ground speed



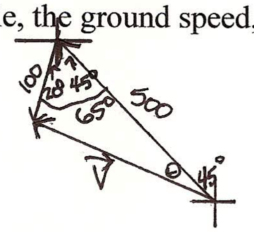
$\dfrac{\sin 45°}{342.3} = \dfrac{\sin\theta}{90}$

$\theta = \boxed{10.7°}$ ← drift angle

$\sin\theta = 0.1859$

course: $270° - 10.7° = \boxed{259.3°}$

Example: A jet is headed northwest with an air speed of 500 mph. The wind is 100 mph with a bearing of 200°. Find the drift angle, the ground speed, and the course of the jet.



$|\vec{v}|^2 = 100^2 + 500^2 - 2(100)(500)\cos 65°$

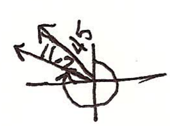
$|\vec{v}|^2 = 217{,}738$

$|\vec{v}| = \boxed{466.6 \text{ mph}}$ ← ground speed

$\dfrac{\sin 65°}{466.6} = \dfrac{\sin\theta}{100}$

$\sin\theta = 0.1942$

$\theta = \boxed{11.2°}$ ← drift angle



Course:

$360° - (45° + 11.2°)$

$= \boxed{303.8°}$